

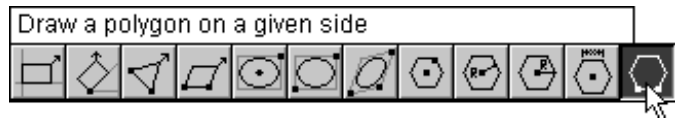
## 12 : PROJECT Tessellations

A tessellation consists of shapes or tiles which fit together with no spaces, allowing the tessellation to be extended indefinitely to cover any area. This simple definition describes an enormous range of designs from a tiled floor to intricate Islamic decoration. If you have seen the repeating patterns created by M. C. Escher you know already about one of the most attractive kind of tessellations, in which the repeating motifs take the form of recognisable forms, birds, fish, reptiles, or human figures.

In this Chapter, you can find out how to generate lots of different tessellations. *2D Design* makes this absurdly easy and you will have no difficulty in creating colourful and attractive tessellated patterns.

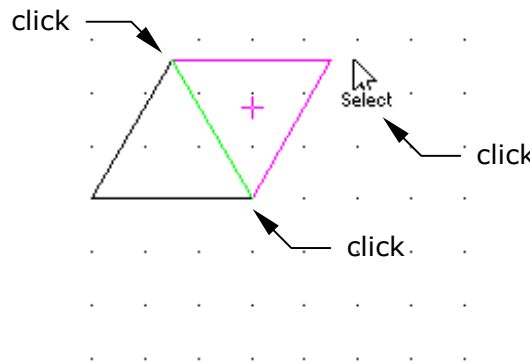
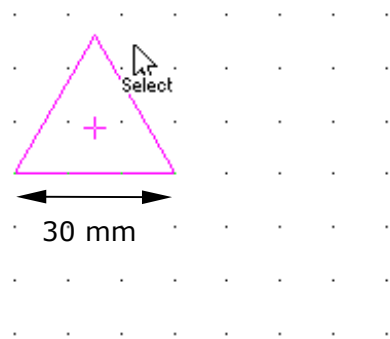
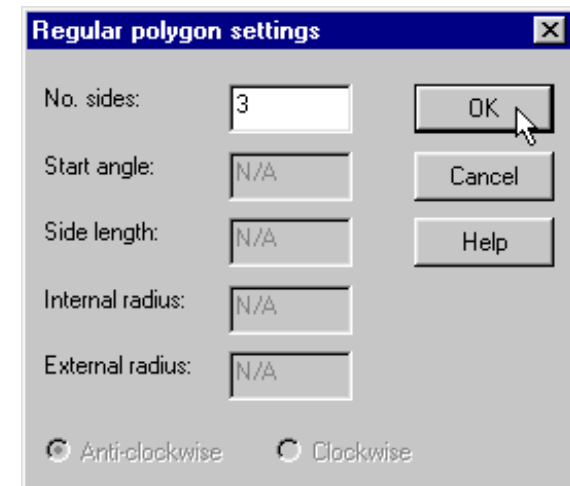
### Which shapes tessellate?

Start a new *2D Design* drawing and change Setup/Drawing/Layout... to ISOA4/Landscape. Click in the drawing aids tool palette to switch Grid Lock on.



Click *and hold* the Shapes button from the draw /edit tool palette to reveal the Shapes fly-out. Move the cursor along by moving the mouse to select 'Draw a polygon on a given side'.

As you release the mouse button, the Regular polygon settings dialogue appears. Change the number of sides to 3 as indicated, and click OK.



Draw a triangle with a base of 30 mm. Click to select the orientation shown in the diagram. Now switch Grid Lock off and switch Attach on. Add a second triangle, clicking at the lower right corner of the first triangle and again at the top. Click to select the orientation as before.

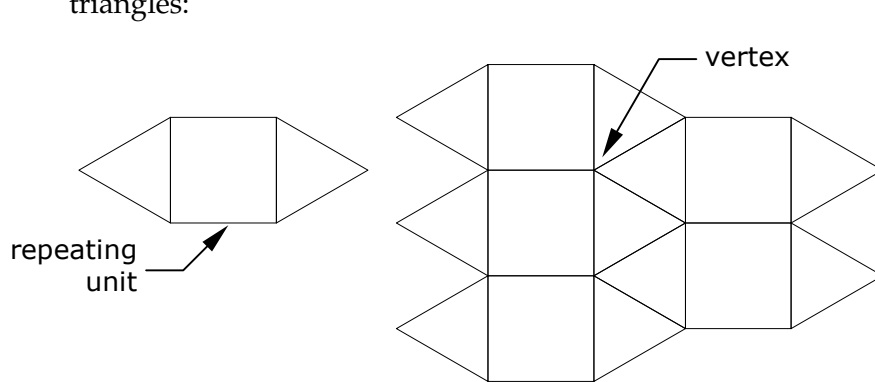
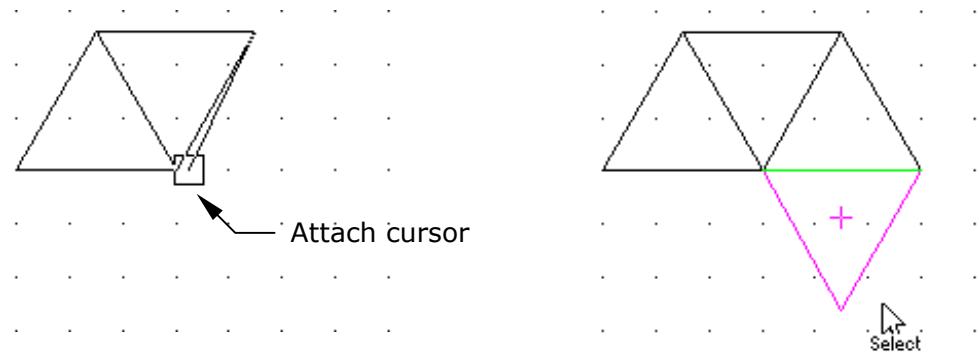
Continue adding triangles using Attach. It is easy to see that the pattern can be extended indefinitely to cover any area. You describe this situation by saying that equilateral triangles *tessellate*.

Right click the Shapes button and change the Regular polygon settings to experiment with polygons with different numbers of sides.

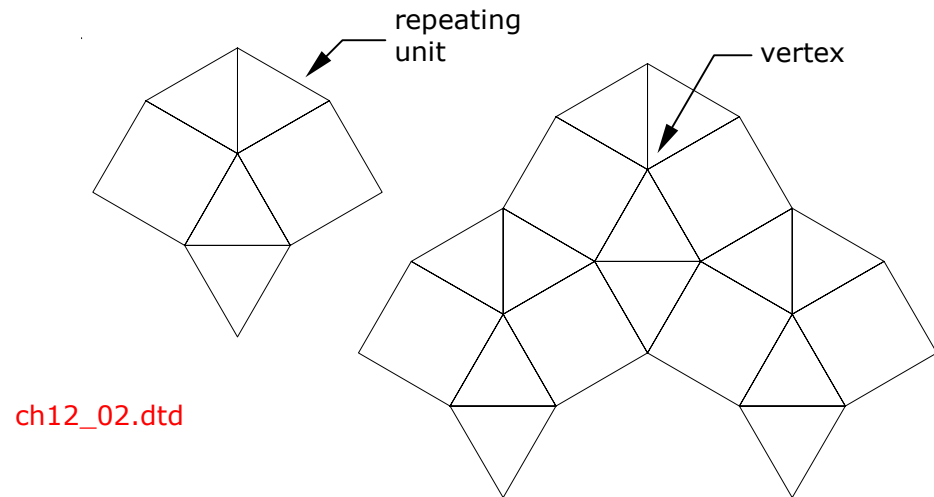
It is obvious that squares will tessellate, but what about pentagons (five sides) and hexagons (six sides)?

In fact, there are just three types of tessellations consisting of regular polygons of a single type. These can be equilateral triangles, squares or hexagons. Where the tessellated shapes meet at a corner, or vertex, the corner angles must add to give  $360^\circ$ . Pentagons do not tessellate because the internal angle at each corner,  $108^\circ$ , does not divide exactly into  $360^\circ$ .

If regular polygons of different types are used in the same tessellation, you can create more patterns. The examples below use squares and triangles:

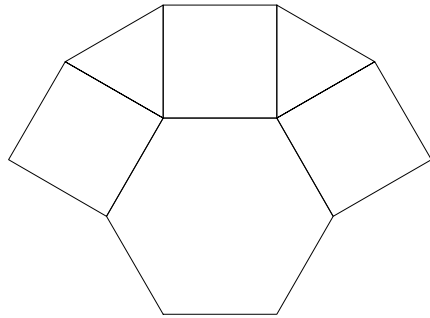


ch12\_01.dtd

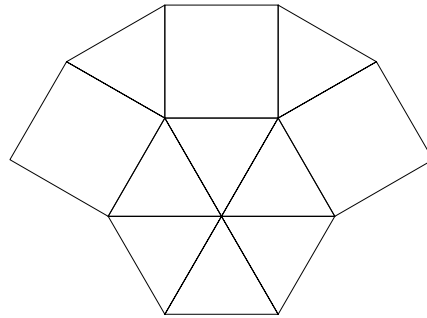


ch12\_02.dtd

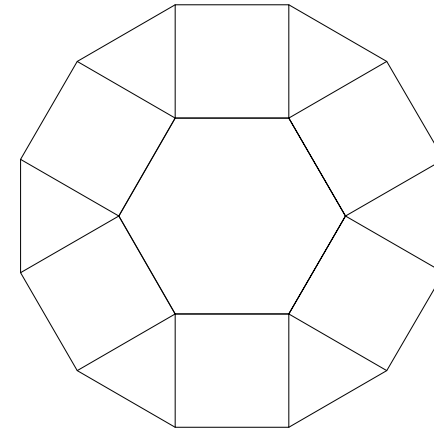
These are *semi-regular* tessellations because the arrangement of tiles at each meeting point, or vertex, is identical. If you look carefully you can see that two squares and three triangles meet at each vertex. Use *2D Design* to draw these tessellations. Draw and group the elements of the repeating unit and use Move/Copy together with Attach to extend the pattern.



ch12\_03.dtd



ch12\_04.dtd

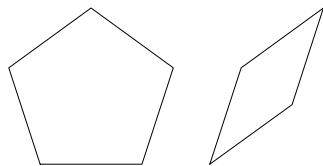


ch12\_05.dtd

Draw and experiment with these subunits consisting of triangles, squares and hexagons. Adding colour enhances the interest of these designs. Look for example at the tessellation [ch12\\_05.dtd](#) from the CD. There are lots of possibilities for creating tessellations using regular polygons.

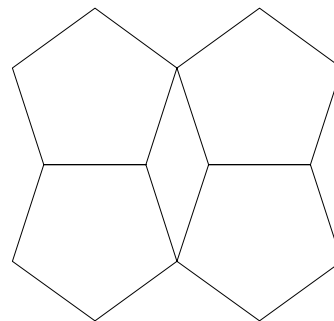
It is true that pentagons do not tessellate on their own, but if diamond-shaped tiles are inserted, you can achieve an attractive result.

tiles required

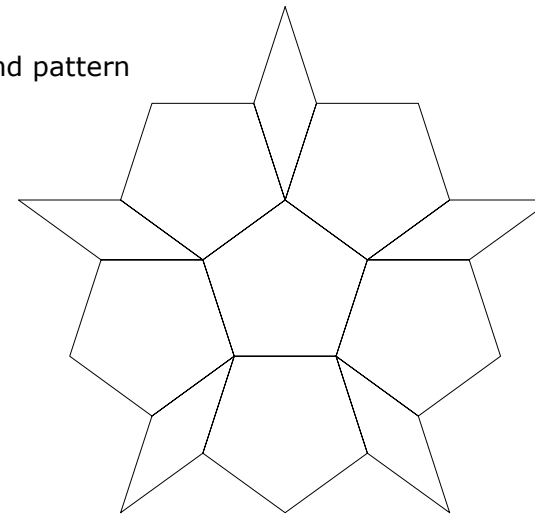


ch12\_06.dtd

first pattern



second pattern



Two quite different tessellations are generated depending on how the first few tiles are arranged. For the first pattern draw two pentagons from the same base and use Edit/Group to join them together as a repeating unit. Use Move/copy and Attach to extend the pattern.

To draw the second pattern, start with a single pentagon and use Attach to draw a new pentagon along each of the five sides. You can continue to add pentagons along the outside edges of this shape indefinitely, see [ch12\\_06.dtd](#), [ch12\\_07.dtd](#). You do not need to draw the diamond-shaped 'fillers' separately: these will appear between the pentagons as you extend the pattern.

### Tessellation of irregular shapes:



Clear the drawing area or start a new drawing. Select 'Draw a triangle' from the Shapes fly-out. Switch off Grid Lock and Step Lock and draw a triangle. The position and shape of the triangle are not important.

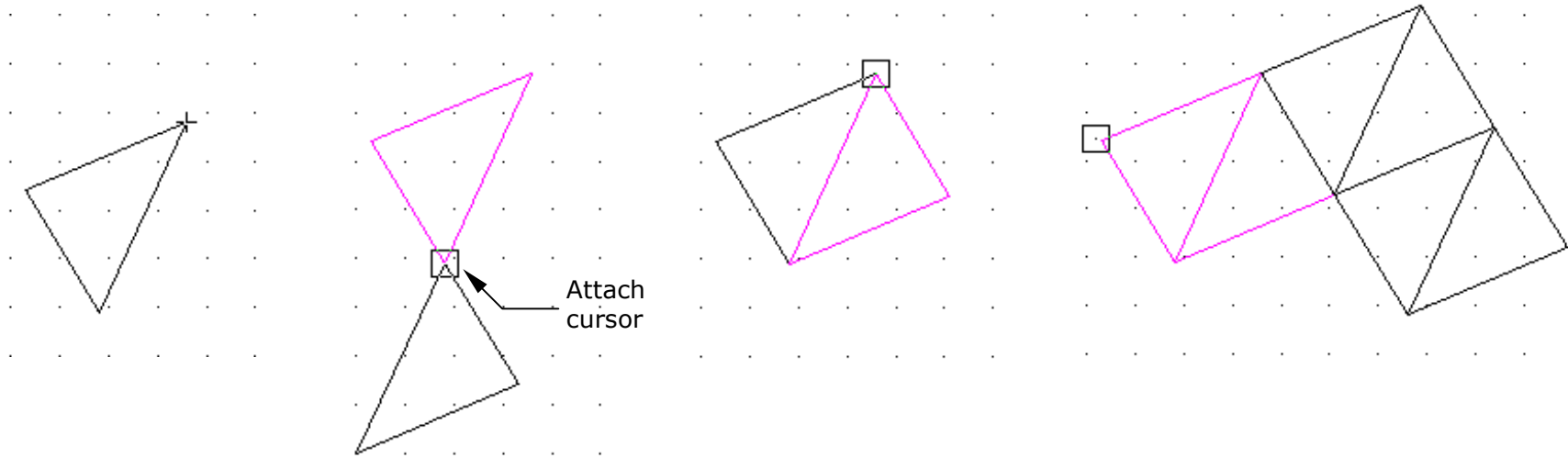
To create a tessellation using your triangle, follow the stages shown in the diagrams. Remember to use the Attach cursor.

*draw the triangle*

*rotate/copy through 180°*

*move/copy next to original*

*group the repeating unit and tessellate*



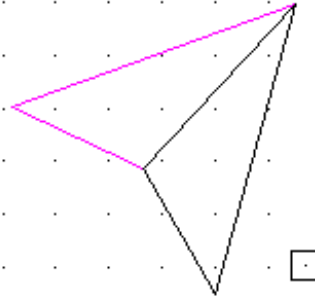
Experiment with other shapes and sizes of triangles to confirm that *any* triangle will tessellate in this way. You can achieve a different effect by making a mirror image of the first triangle, instead of rotating through 180°.

The drawing sequence is shown on the opposite page. Repeat the process creating the initial mirror image on a different side of the triangle.

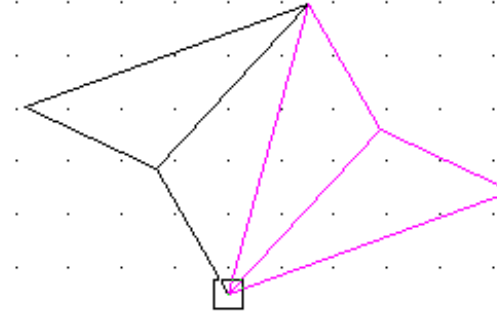
*draw triangle*



*mirror image along any side*



*rotate/copy through 180°*



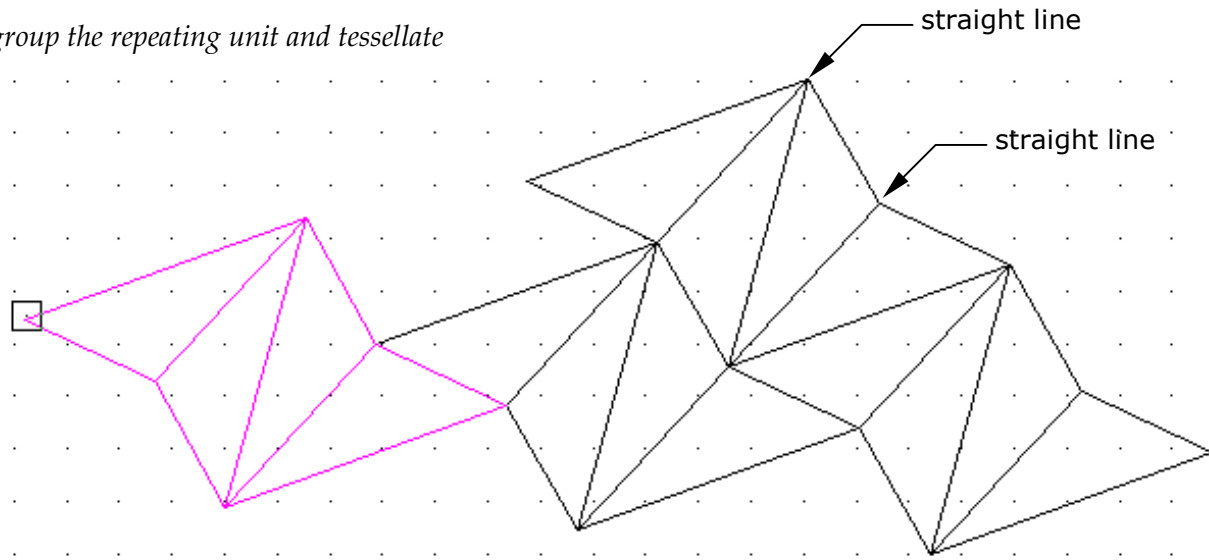
The drawing is completed with Grid Lock and Step Lock switched off.

You will find that three quite different patterns result depending on which side of the triangle you use to provide the mirror axis.

In designs generated in this way, one edge of each triangle will line up with the edge of the next triangle to form straight lines passing through the tessellation.

Experiment until you are confident about the drawing methods used in creating these designs.

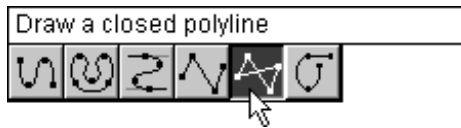
*group the repeating unit and tessellate*



In this section, you have discovered that any triangle will tessellate. What about irregular four-sided shapes, called quadrilaterals?

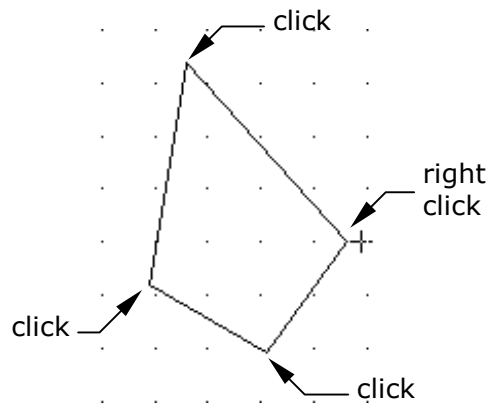


Start a new drawing. Locate the Path button from the draw/edit tool palette and click and hold to reveal the Path fly-out. Move the cursor along and release the mouse button to select 'Draw a closed polyline'. Switch Grid Lock/Step Lock off and switch Attach on.

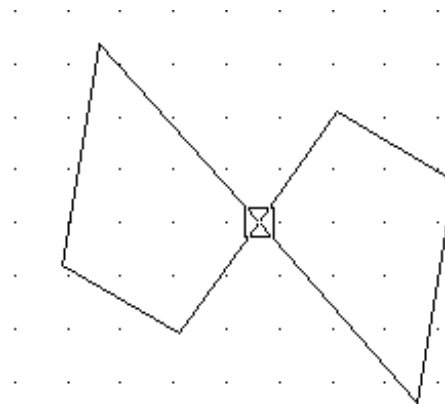


The Path options give you an easy way of creating line segments and bezier curves which are linked so that they behave as a single entity. Draw an irregular quadrilateral shape by clicking to define the first three corners and right-clicking to close the shape:

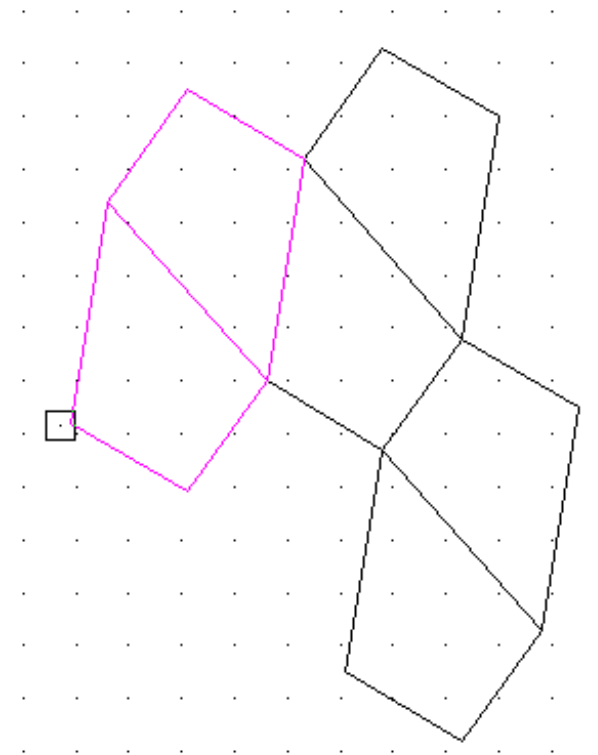
*draw the quadrilateral*



*copy/rotate through 180°*



*move, group the repeating unit and tessellate*



As you can see, an irregular quadrilateral can easily be made to tessellate. Experiment with other shapes to confirm that *any* quadrilateral will tile together in this way.

Experiment by mirror-imaging the quadrilateral. This does not give you a tessellating shape, as you will soon discover.

The techniques you have used so far in this Chapter are fundamental for the creation of tessellations and will now be explained more precisely.

There are just three distinct operations which can be used to locate a new tile in a tessellated pattern. These are:

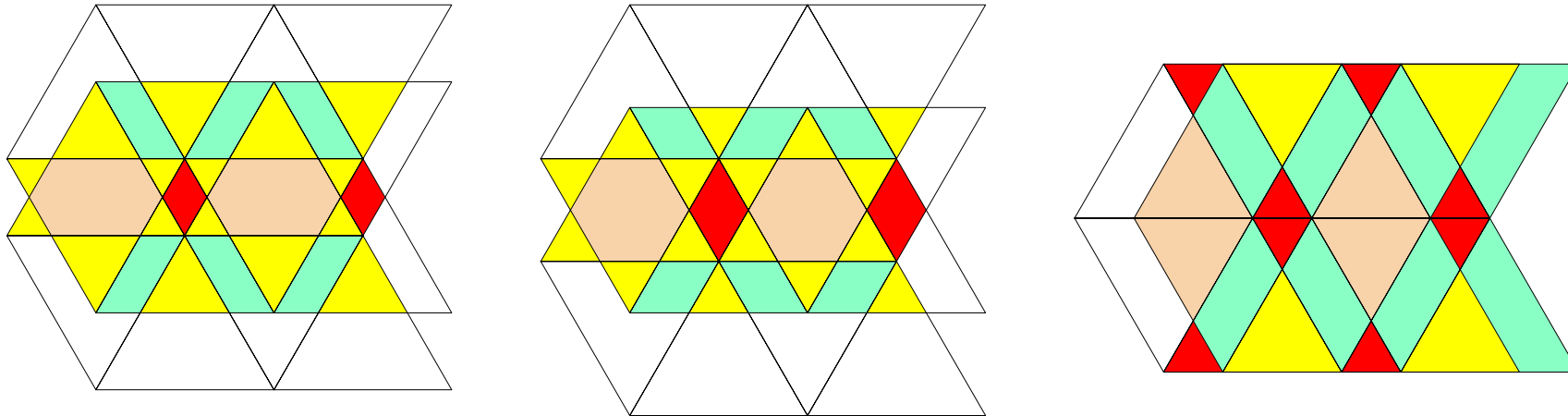
1. **TRANSLATION:** This is a lateral or diagonal movement compared with an existing tile. The new tile is not rotated.
2. **ROTATION:** The tile is turned around its centre point before being placed in its new location.
3. **REFLECTION:** A mirror image of the tile is created. This is equivalent to turning a tile over so that its lower surface is now uppermost.

Look back at the tessellations you have drawn and check for examples of these operations, individually and in combination.

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## Generating new tessellations:

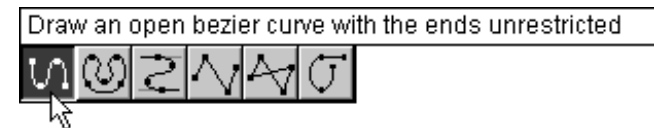
A straightforward method of creating a new tessellation is to overlap tiles from one of the simple tessellations described earlier. The examples below are derived by overlapping triangular grids, [ch12\\_08.dtd](#).



Alternatively, new patterns can be developed by separating the tiles, leaving spaces which can be filled with other tiles. This theme is developed in [ch12\\_09.dtd](#). A tremendous variety of geometric designs can be created in this way. You can explore these possibilities easily using *2D Design*.

A rather different approach is to use the grid intersections to define points of symmetry for the tessellation of irregular shapes. The best way to understand this idea is to create an irregular tessellation of your own. Clear the *2D Design* drawing area or start a new drawing. With Grid Lock switched on, draw an equilateral triangle on a base of 30 mm, as illustrated on the next page.

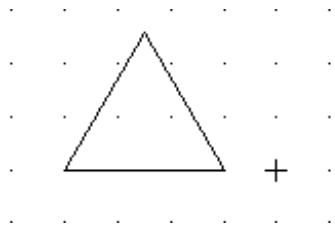
Switch Grid Lock off and switch Attach on. Select the open bezier option from the Path fly-out. Click at the top corner of your triangle to start the curve. Click once or twice on either side of the left hand side of the triangle and right click to attach the curve to the left hand corner of the triangle.



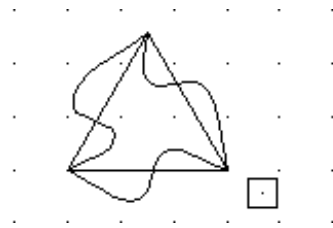
Depending on the shape you want to achieve, you may want to switch Attach off temporarily before switching it on again to click at the corner of the triangle.

Draw bezier curves along the other two sides of the triangle in just the same way. The exact shape of these curves is not important, although they *must* be attached to the corners of the triangle. You will be able to make adjustments later if required.

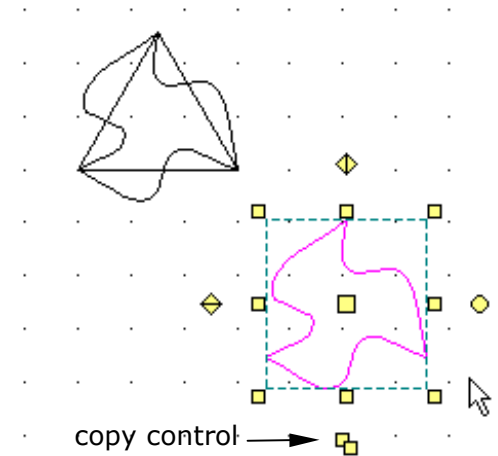
*draw triangle*



*draw a bezier curve along each side*



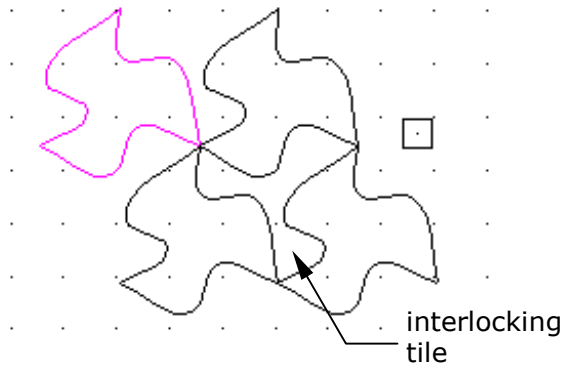
*group and copy the bezier outline*



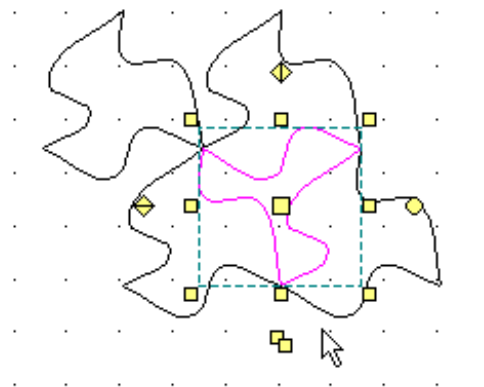
Next, use the Select cursor together with the **SHIFT** key to highlight the three bezier curves. Copy the curves by clicking the copy control and then Group them from the Edit menu.

This new shape is one of the tiles for the tessellation. Use Attach together with Move/copy from the Transform fly-out to copy the tile, matching the appropriate corner positions. As you place the tiles, the shape of a second interlocking tile becomes clear.

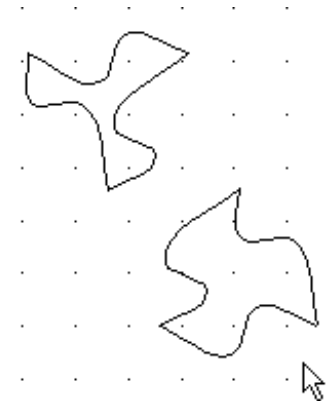
*first few tiles*



*ungroup and select interlocking tile*



*the two tiles needed for the tessellation*



Ungroup the existing tiles and select the curves which make up the interlocking tile. You can group and copy this tile in the same way as the first. Extend the tessellation, placing both types of tile using Attach and Move/copy, see [ch12\\_10.dtd](#).

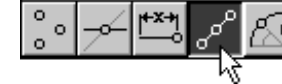
Tiles created in this way are *translated* into new positions: No rotation or reflection is required.

To create a rotation tessellation, you need to follow a different method. Start a new drawing, switch Grid Lock on and draw an equilateral triangle on a base of 30 mm. Switch Grid Lock off and switch Attach on.

Select 'Draw construction points which divide a given object' from the Const. Points fly-out. Check that No. of divisions is set to '2' and click OK. Click anywhere along the left hand side of the triangle to locate a construction point at its mid-point.

For this example, select the open polyline option from the Path fly-out. Attach the beginning of the path to the top of the triangle and zig-zag once or twice before attaching the end of the path to the *construction point*.

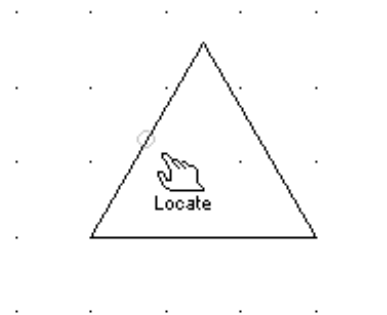
Draw construction points which divide a given object



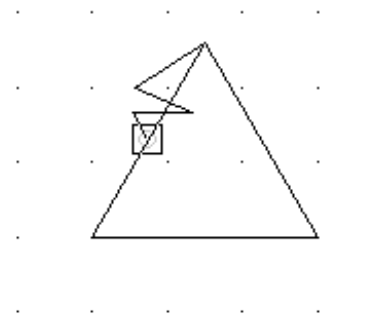
Draw an open polyline



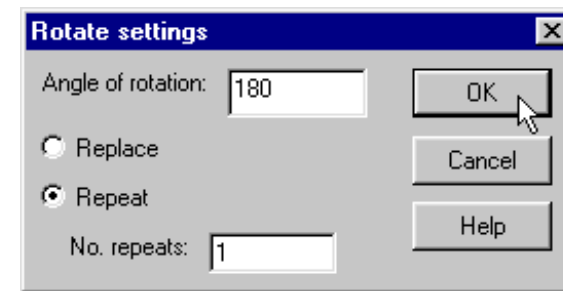
*locate construction point*



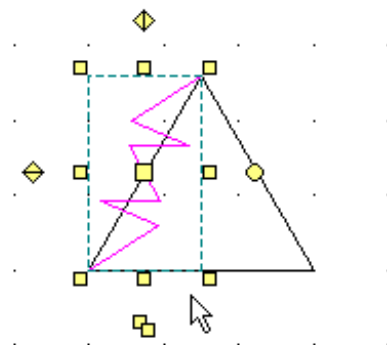
*draw polyline segments*



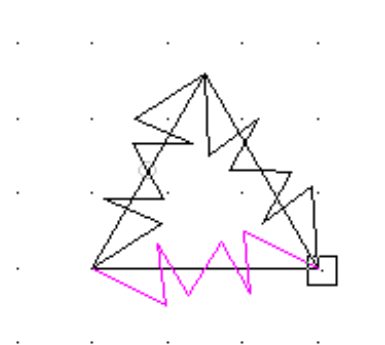
*copy and rotate 180°*



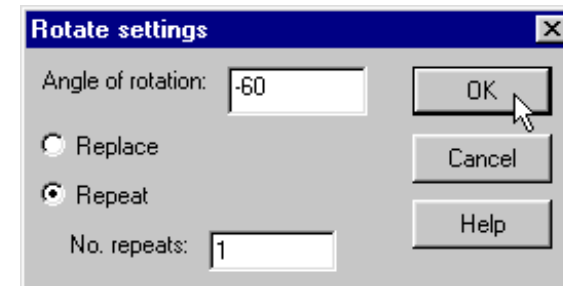
*make path*



*rotate and copy around triangle*

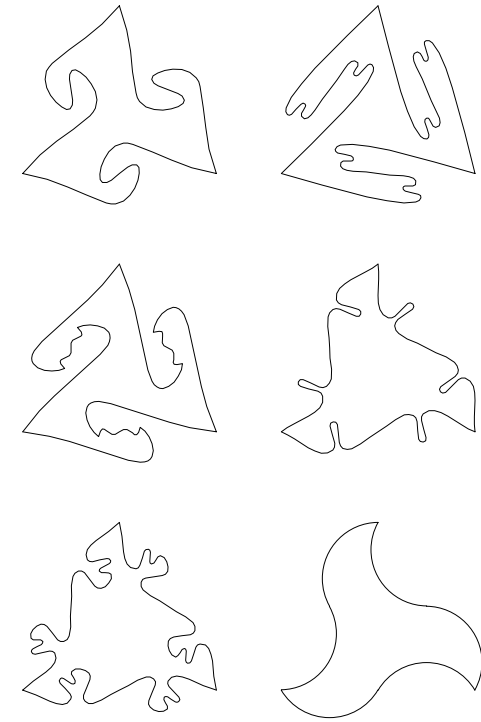
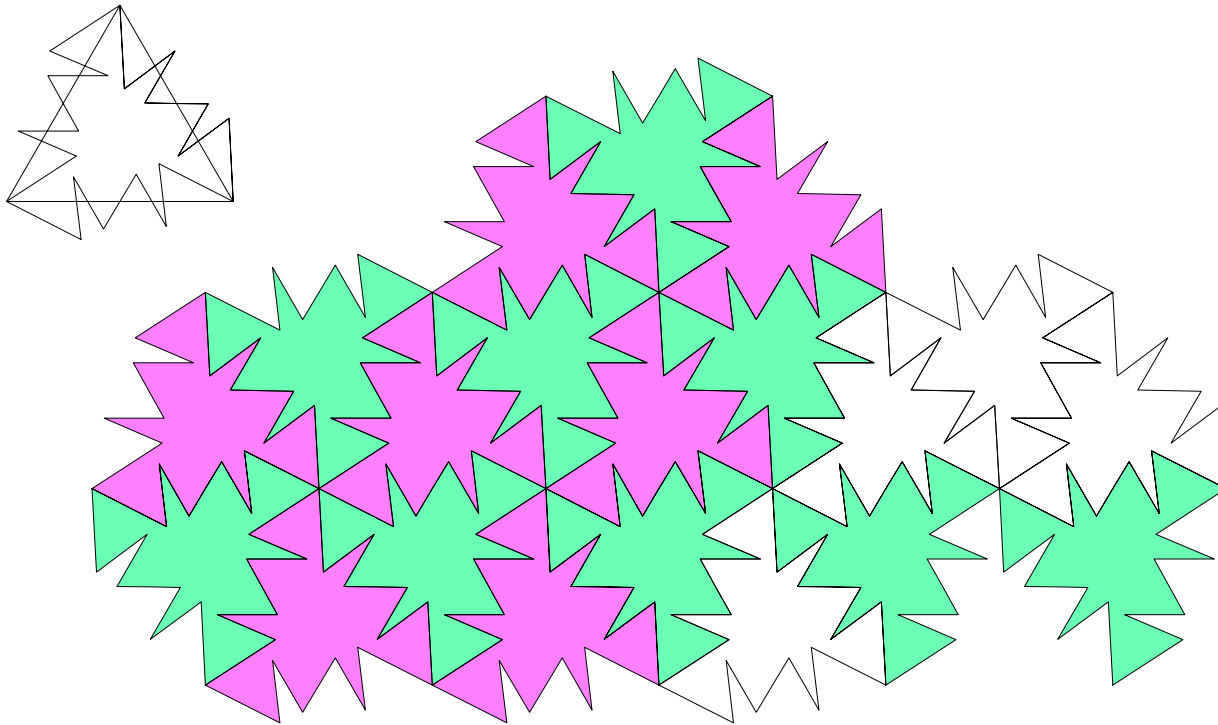


*clockwise rotation by -60°*



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Following the steps illustrated in the diagrams will give you a zig-zag outline which you can group and copy. Create the tessellation using rotate/copy. Increase the No. of repeats setting if you want the design to grow more quickly.



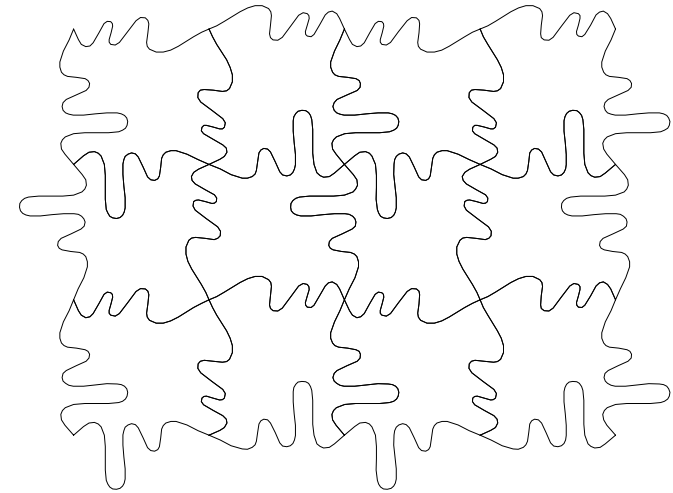
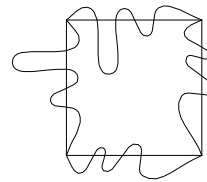
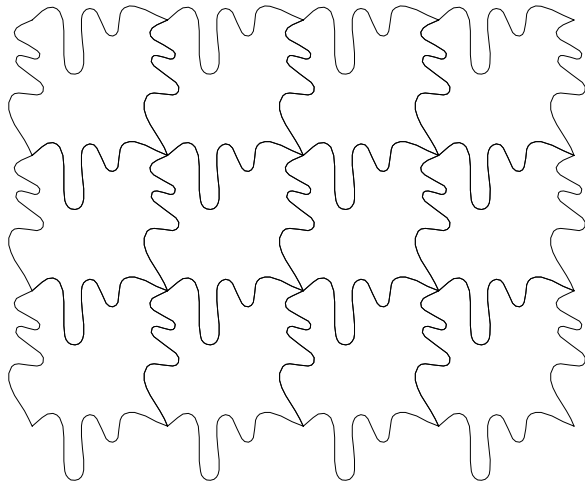
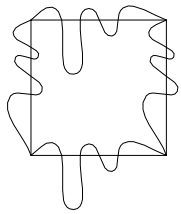
*rotation tessellation: [ch12\\_11.dtd](#)*

*alternative rotating tiles*

Experiment with your own rotating tiles and look at the tessellations created from the alternative tiles, [ch12\\_12.dtd](#), [ch12\\_13.dtd](#) and [ch12\\_14.dtd](#).

The same techniques can be used with square and hexagonal tiles. For a translation tessellation to work, you must copy the line segment or bezier curve from the top of a square to the bottom and copy from one side of the square to the other. For a rotation tessellation, copy and rotate from the top to the left hand side of the square and from the bottom to the right hand side.

In the example opposite, the translation tessellation is easily recognised because all the tiles have the same orientation. In the rotation tessellation, the tiles can be oriented in any of four different ways. Adjacent tiles are rotated 90° with respect to each other.



*translation*

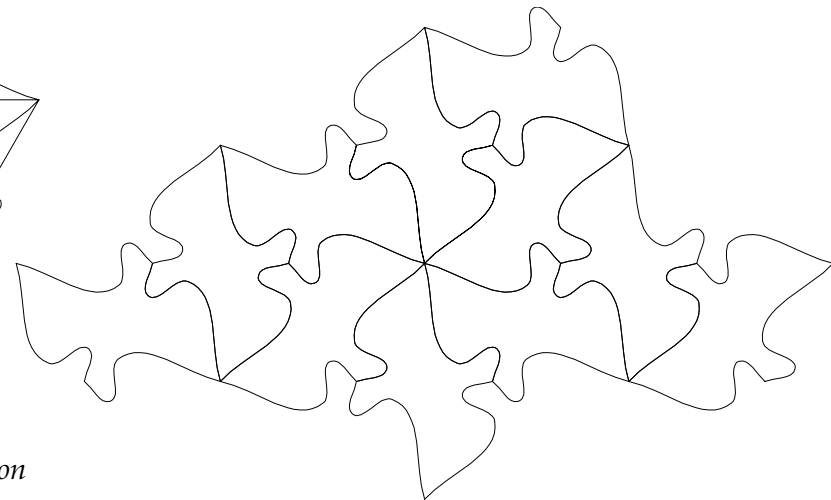
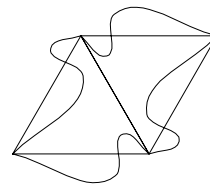
*rotation*

With hexagons, you can control the appearance of the tessellation in just the same way. Look in the resources section for Escher designs based on hexagon tessellation.

A useful four-sided shape called a rhombus can be made from two equilateral triangles.

Draw a bezier curve along the left side of the rhombus and rotate/copy by  $-60^\circ$  to locate the curve along the bottom. Rotate/copy again by  $-120^\circ$  to copy the curve to the right side and then by  $-60^\circ$  to complete the outline. (The minus sign indicates rotation in a clockwise direction.)

Group the outline and rotate/copy to create the tessellation. A point where tiles meet is called a vertex. The number of tiles which meet at a vertex is either three or six. How many vertices are there of each type?



*rhombus tessellation*

Use *2D Design* to find out how the sides of the rhombus should relate so that the tiles build together to make a translation tessellation.

## Escher tessellations:

M. C. Escher was a Dutch graphic artist with a life-long fascination with periodic designs and perspective. He created hundreds of tessellations in which the repeating motifs were natural forms including birds, fish, reptiles, insects and human figures. His designs are intricate and attractive. You can find out more about his work by following up the references in the resources section of this Chapter.

The fish motif shown on the opposite page was built up in stages, as indicated in the diagrams.

The tessellation is based on a rectangle, drawn 40 x 20 mm. A path representing the fins is drawn along the top of the rectangle, using **Attach** to locate the corner points. This path is copied as a mirror image to the bottom of the rectangle, to represent the lower fins and suggest the outline of the body.

Remember that you can edit the shape of any bezier by clicking **Start Edit** from the edit tool palette. This means that you can modify the top outline to achieve the result you want.

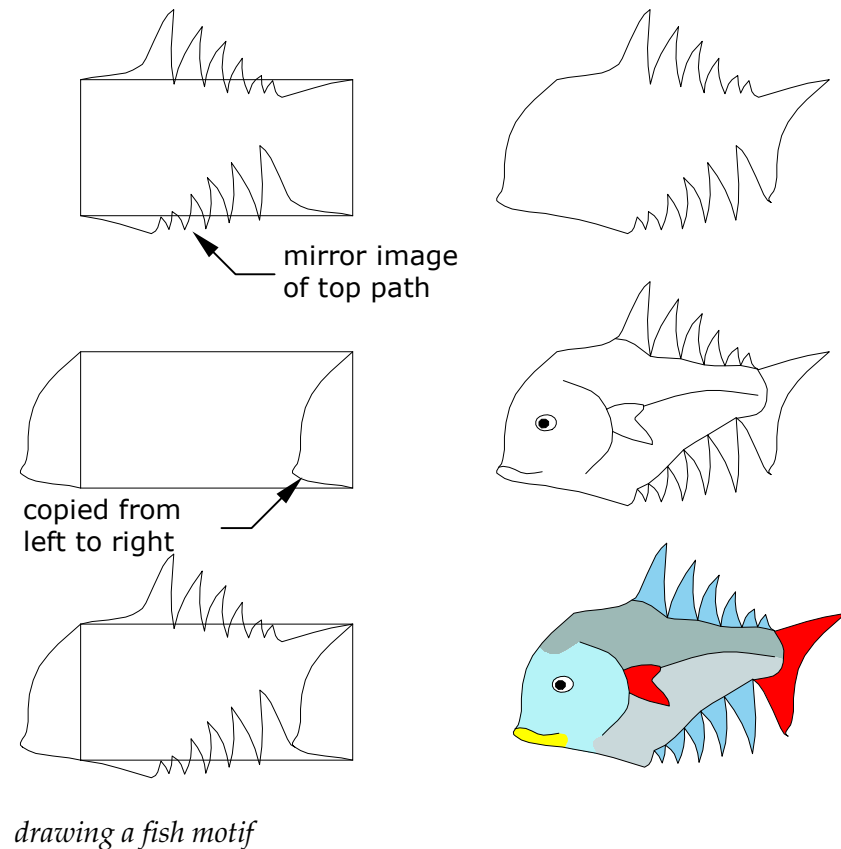
Next, the bezier representing the head of the fish is added. This is copied from the left side of the rectangle to the right *without* rotation or reflection.

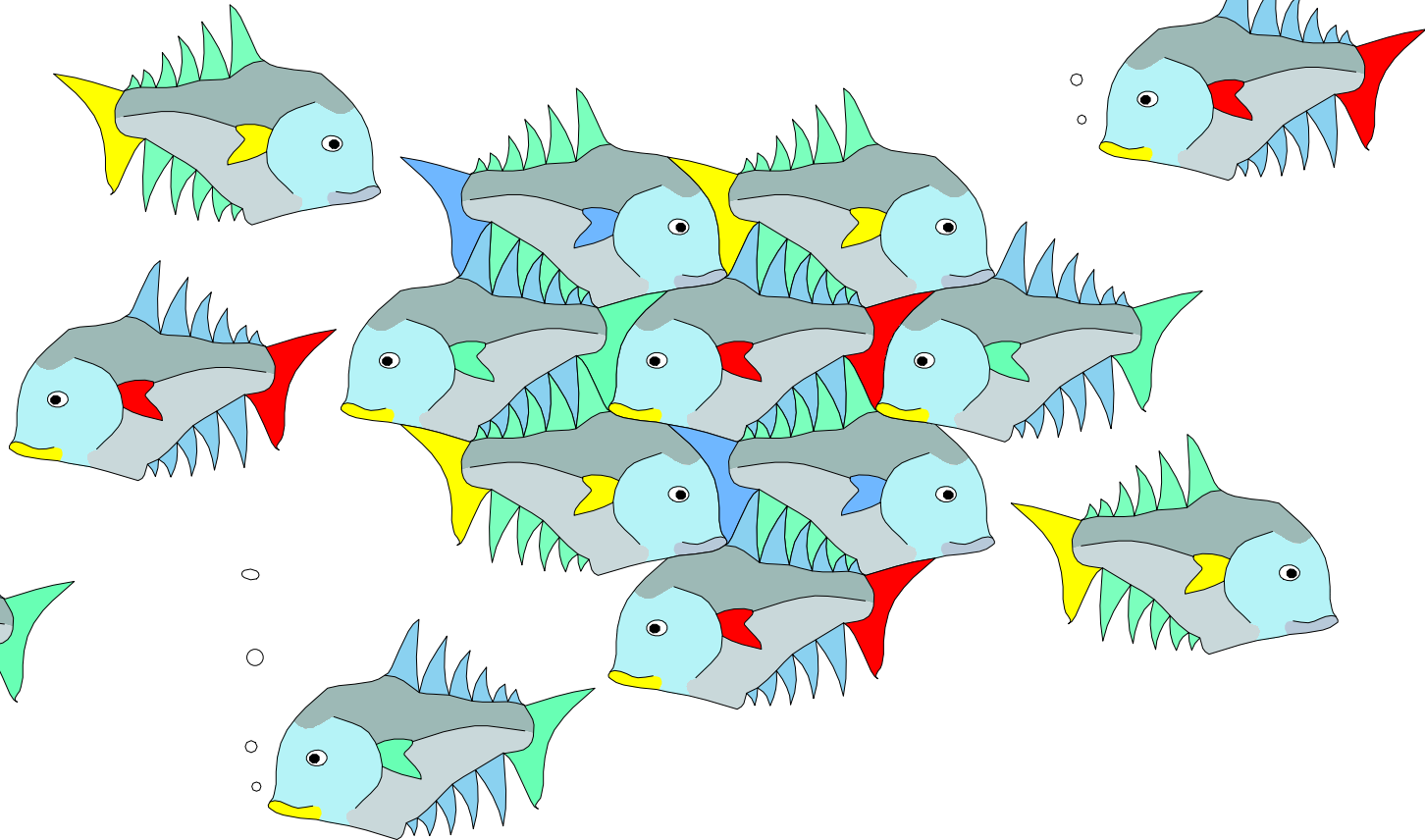
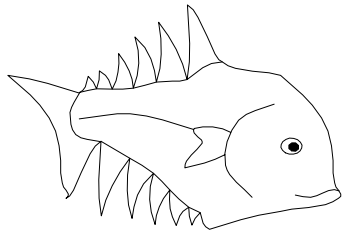
The outline of the fish within the rectangle is obvious. Tidy up the outline by deleting the rectangle and the short line segment attached to the bottom of the tail. Use **Make path** from the **Edit** menu to join the line segments together as a single entity.

Line details and colour fills are added to complete the fish motif.

To generate the tessellation, the fish are translated from left to right along the same row. To locate the fish on the next row, you need to mirror image, or reflect the motif. This is a favourite Escher transformation, known as glide reflection.

Many Escher tessellations have a simple underlying geometry. The difficulty comes in analysing exactly how the tessellation was produced. The [Totally Tessellated](#) Internet site included in the resources section gives a particularly clear explanation of Escher's basic techniques. It is not easy to draw convincing Escher tessellations. The more you study his work, the more impressed you will be.



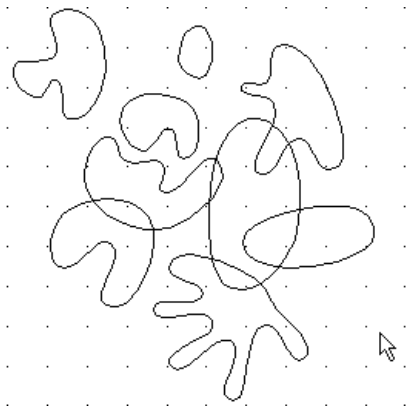


ch12\_15.dtd

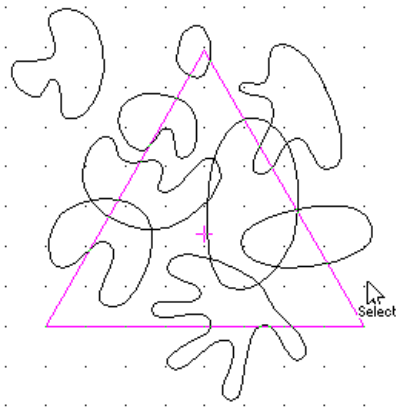
## Kaleidoscope tessellations:

Unlike Escher tessellations, kaleidoscope tessellations are really easy to draw. Start a new drawing file, switch Grid Lock and Step Lock off and select the closed bezier curve option from the Path fly-out. Draw 'random' bezier outlines somewhere near the middle of the drawing area. Make the shapes overlap and move them around until they form a pleasing pattern.

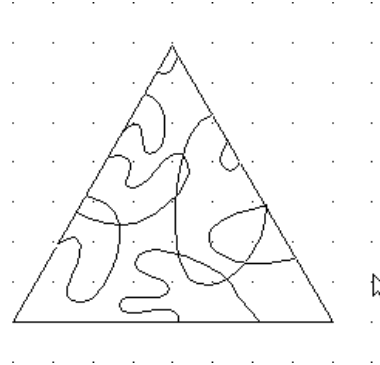
*bezier outlines*



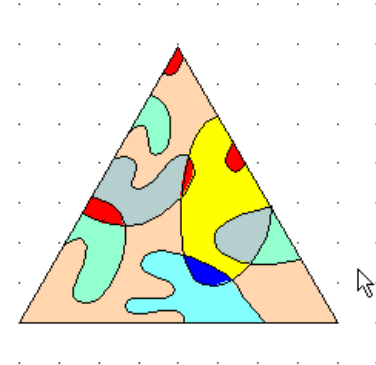
*add equilateral triangle*



*trim shapes*

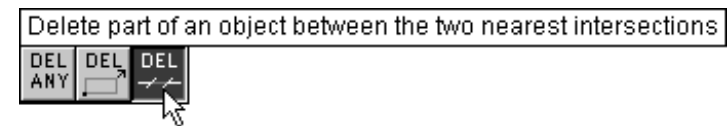


*add colour and group*



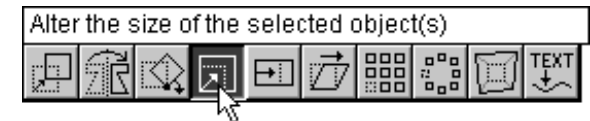
Switch Grid Lock on and draw an equilateral triangle using 'Draw a polygon on a given side' from the Shapes fly-out. The triangle should enclose most of the shapes and, with Grid Lock switched off once more, you can move the triangle around and/or resize it to give the best effect.

Use 'Delete between intersections' to trim the parts of the bezier shapes outside the triangle. Now add colour fills to the different areas inside the triangle. You can choose any colour you like from the Fill colour and Custom colour dialogues.



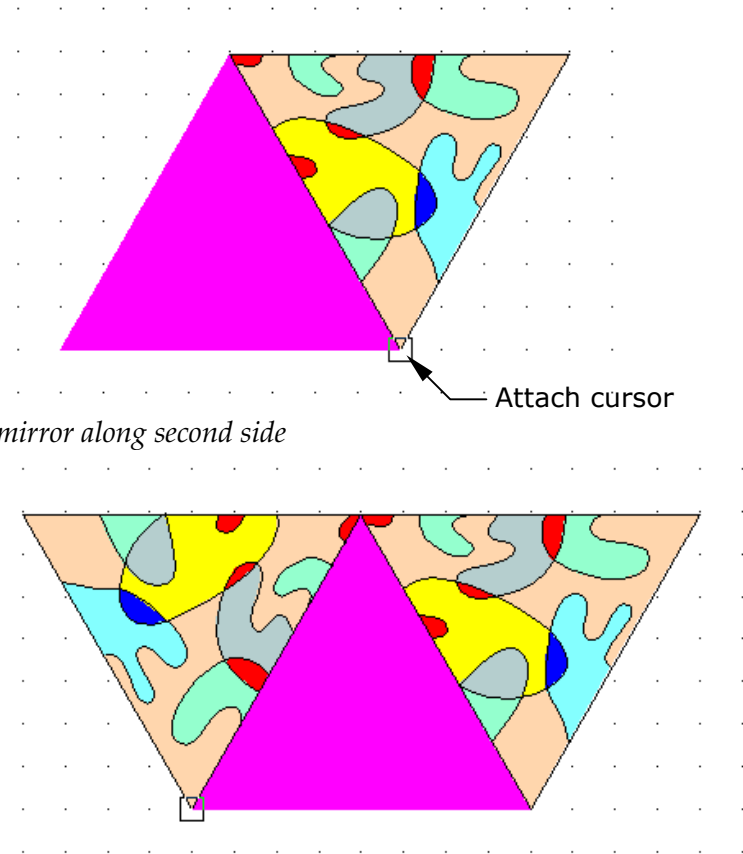
When you are happy with your triangle, highlight the complete design using the Select cursor and Group from the Edit menu. Switch Attach on and mirror the triangle first along one side and then the other. Select all three triangles and mirror again to generate a hexagon.

Group the hexagon. Your design is probably too large in relation to the drawing area. Reduce its size using the appropriate option from the Transform fly-out. The hexagon can be tiled off to cover any area using Move/copy together with Attach.

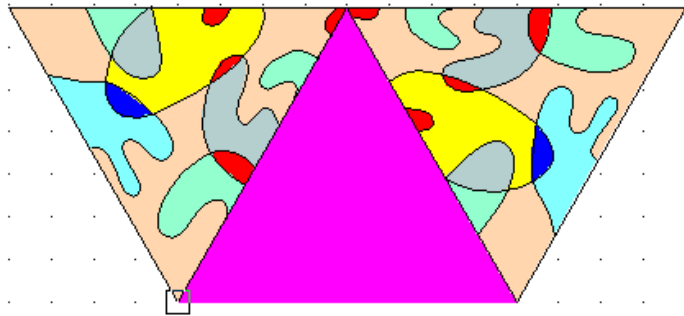


The tessellation from this particular design is saved on the CD as [ch12\\_18.dtd](#). Interesting patterns develop where hexagons meet.

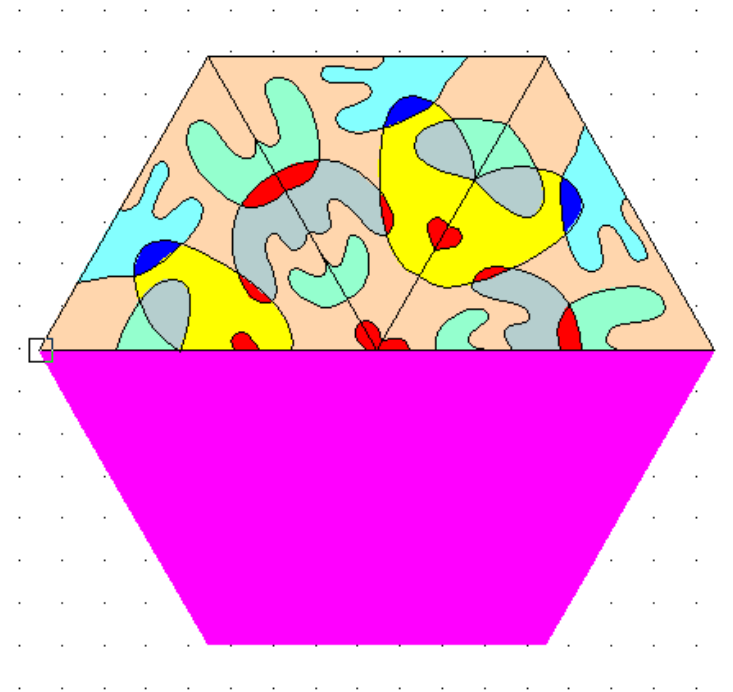
*mirror along first side*



*mirror along second side*



*mirror again to generate hexagon*



ch12\_18.dtd

There are several other kaleidoscope tessellations for you to look at, [ch12\\_19.dtd](#) - [ch12\\_24.dtd](#). These are just examples and it is much more fun to create your own.

Resources:

The Project Chapters of *Discovering CAD* included suggestions of books and Internet resources which will give you even more ideas.

*Mosaic and Tessellated Patterns, How to Create Them*, John Willson, Dover Publications, Inc., New York, 1983, ISBN 0-486-24379-6.

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*Polysymetrics, the Art of Making Geometric Patterns*, June Oliver, Tarquin Publications, Stradbroke, Diss, NORFOLK IP21 5JP, 1998, ISBN 0-906212-09-X.

*The Magic Mirror of M. C. Escher*, Bruno Ernst, Tarquin Publications, Stradbroke, Diss, NORFOLK IP21 5JP, 1985, ISBN 0-906212-45-6.

An excellent introduction providing an overview of Escher's enigmatic life and the whole range of his work.

*M. C. Escher Kaleidocycles*, Doris Schattschneider and Wallace Walker, Tarquin Publications, Stradbroke, Diss, 1982, NORFOLK IP21 5JP, ISBN 0-906212-28-6.

17 models of Escher's graphics and the story of how they were devised. The magic of Escher's work is here extended and developed into a colourful set of three dimensional models you can make for yourself.

*M. C. Escher, Visions of Symmetry*, Doris Schattschneider, W. H. Freeman and Company, 20 Beaumont Street, Oxford OX1 2NQ, ISBN 0-7167-2352-2.

'How did he do it?' This beautifully produced and comprehensive book is the author's answer.



Search for these books on the Internet at [amazon.co.uk](http://amazon.co.uk) or [amazon.com](http://amazon.com), or order through your bookseller.

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#### CD-ROM

*Escher Interactive: Exploring the art of the Infinite*, © 1996 Eyeware Interactive BV and Cordon Art BV, Thames & Hudson, ISBN 0-500-10016-0.

CD-ROM for *Windows*. Check that your system is compatible. An excellent resource including more than 400 graphic works, a tessellation workshop, animation, morphing, impossible puzzles and more.

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#### Internet Resources



<http://library.thinkquest.org/16661/index2.html>

A marvellous resource giving an easy to understand explanation of Escher tessellations and comprehensive information on other sorts of tessellation.

*Tessellation Tutorials*, Susan Alejandre: <http://forum.swarthmore.edu/sum95/suzanne/tess.intro.html>

*The Geometry Junkyard: Tiling*: <http://www.ics.uci.edu/~eppstein/junkyard/tiling.html>

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